

Superdiffusion in the topological metal

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We develop a non-perturbative theory to study large-scale quantum dynamics of Dirac particles in disordered scalar potentials (the so-called “topological metal”). For general disorder strength and carrier doping, we find that at large times, superdiffusion occurs. I.e., the mean squared displacement grows as $\sim t \ln t$. In the static limit, our analytical theory shows that the conductance of a finite-size system obeys the scaling equation identical to that found in previous numerical studies. These results suggest that in the topological metal, there exist some transparent channels – where waves propagate “freely” – dominating long-time transport of the system. We discuss the ensuing consequence – the transverse superdiffusion in photonic materials – that might be within the current experimental reach.

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The past years have witnessed that graphene [1, 2] and topological insulators [3] become an exciting frontier of condensed matter physics. A characteristic feature common to these novel materials is the emergence of the massless Dirac particle. For example, it appears on the surface of three-dimensional strong topological insulators such as Bi_2Se_3 and Bi_2Te_3 [3–5]. This particle displays helical spin (that accounts for the sublattice structure in graphene) structure in momentum space and is “relativistic”: the energy dispersion is linear. On one hand, the discovery of the Dirac particle in these materials is fostering conceptual developments in physics, notably the Klein tunneling in condensed matter systems [6]; on the other hand, with many striking properties, the Dirac particle may find considerable potential applications in new electronic devices.

For experiments and practical applications, the interplay between Dirac particles and disorders is a key issue [7]. A complete theory must include the description of quantum dynamics of the Dirac particle in disordered environments. This is a subject largely unexplored. Yet, there have been increasing evidences indicating that rich dynamic phenomena might occur in this system. Indeed, in the absence of disorders, experimental and theoretical studies (see Refs. [8, 9] and references therein) have shown that the Dirac particle has already exhibited interesting dynamic behavior. In addition, being in the same symmetry class notwithstanding, a disordered Dirac particle system and a normal metal with disordered spin-orbit coupling have transport properties of profound differences [10–12]. It is thereby conceivable that in the presence of disordered potentials, the interplay between wave interference and the relativistic energy dispersion may lead to even more interesting dynamic behavior and eventually, to unusual electric and optical properties.

The purpose of this Letter aims at a systematical and analytical investigation of this subject. Specifically, we shall focus on the two-dimensional system with a single Dirac valley and subject to scalar disordered potentials

$V(\mathbf{x})$, the so-called “topological metal” [10]. For this system, the quantum dynamics is characterized by a two-component spinor, Ψ , obeying (We set $\hbar = 1$)

$$i\partial_t\Psi = \hat{H}\Psi, \quad \hat{H} \equiv -iv(\sigma^x\partial_x + \sigma^y\partial_y) + V(\mathbf{x}). \quad (1)$$

Here, v is the Fermi velocity and σ^i , $i = 0, x, y, z$ stands for the Pauli matrices. The effective time-reversal symmetry, i.e., $\sigma^y\hat{H}^*\sigma^y = \hat{H}$, brings the system to the symplectic symmetry class. Below we develop a non-perturbative theory to study quantum dynamics (1) at large scales.

Armed with this analytical theory, we find that *for general disorder strength and carrier doping*, the quantum transport of this system exhibits certain “anomalies”. I.e., (i) at large times, quantum *superdiffusion* occurs: the mean squared displacement – characterizing the expansion of wave packets – is given by

$$\delta\mathbf{x}^2(t) \xrightarrow{t \rightarrow \infty} \frac{1}{4\pi^2\rho_{\epsilon_F}} t \ln t, \quad (2)$$

where ρ_{ϵ_F} is the density of states at the energy ϵ_F . It suggests that such a topological metal behaves as a “transition” from Ohmic ($\delta\mathbf{x}^2 \sim t$) to perfect ($\delta\mathbf{x}^2 \sim t^2$) metals. The $t \ln t$ behavior, though of pure quantum origin (as we will see shortly), is akin to a classical transport anomaly discovered long ago [13]. Experiences in studies of classical superdiffusion [13] then suggest that the finding of quantum superdiffusion might have far-reaching impacts on many scientific branches including statistical physics, condensed matter physics, nonlinear dynamics, and pure mathematics. We notice that sufficiently away from the Dirac point, this expression is *universal*: it does not depend on disorders at all, as ρ_{ϵ_F} converges to its clean limit. In contrast, near the Dirac point, this expression is non-universal because ρ_{ϵ_F} is governed by disorders. (ii) In the static limit, our analytical theory shows that for a disordered sample of size L , the conductance $g(L)$ follows

the scaling equation,

$$\frac{d \ln g}{d \ln L} = \frac{1}{\pi g}, \quad (3)$$

irrespective of g . This is consistent with numerical findings [2, 10–12]. Remarkably, unlike a normal metal with disordered spin-orbit coupling, this system does not exhibit two-dimensional Anderson transition. The nature of anti-localization in topological metals is currently under intense investigations [10, 14].

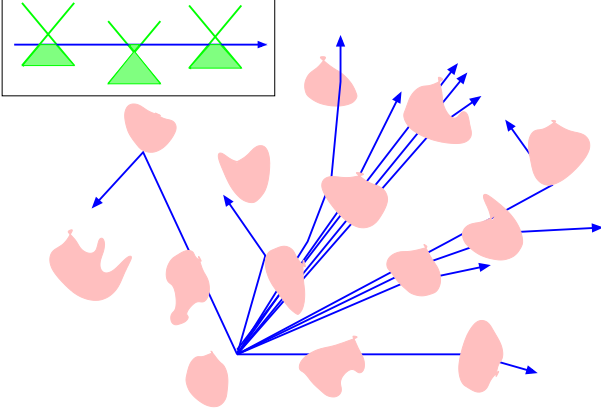


FIG. 1: Multiple scattering of a Dirac particle by disordered potentials (pink). Inset: Klein tunneling.

Let us first discuss a possible physical picture underlying Eqs. (2) and (3) and give their “intuitive” derivation. In doing so, we will see that (i) long-time transport of the system is dominated by transparent channels where waves propagate “freely”; and (ii) the full quantum diffusion coefficient (including multiple scattering effects) displays logarithmic singularity.

We shall focus on the Dirac point ($\epsilon_F = 0$). In this case, the Klein tunneling [6] generally occurs (Fig. 1, inset): upon hitting a potential barrier, a particle may be converted into a hole and subsequently undergoes perfect transmission; similar scenario may happen to a hole. On general grounds, we expect (Fig. 1) that assisted by the Klein tunneling, particles acquire a significant probability of passing “freely” through disordered potentials before deflecting from the incidence direction, leaving a “free” flight path (much) longer than the distance between two nearest disordered potentials. In the extreme case, they keep moving along the incidence direction and pass freely through all the disordered potentials encountered, leaving an infinitely long free flight path (“transparent channels”). Thus, multiple scattering leads to a length distribution of the free flight path.

The diffusion coefficient scales as $\sim v^2 \gamma^{-1}$, where the scattering rate γ is inversely proportional to the length of the free flight path defined above and thereby includes multiple scattering effects. Averaging $v^2 \gamma^{-1}$ with respect

to the scattering rate distribution $P(\gamma)$ gives the long-time asymptotic of the diffusion coefficient, $D(t^{-1} \rightarrow 0) \sim v^2 \int d\gamma P(\gamma) \gamma^{-1}$. Because of $P(0) \sim (\rho_{\epsilon_F} v^2)^{-1}$, the integral over γ suffers logarithmic divergence and is dominated by $\gamma \sim 0$. That is, long-time transport of the system is dominated by the (almost) transparent channels, i.e., $\gamma \rightarrow 0$. Since the free flight path cannot be longer than vt at finite time, i.e., $v\gamma^{-1} \leq vt$, the scattering rate has a lower bound, t^{-1} . (The upper bound is of minor importance as we are interested in the time-dependent behavior.) Taking this into account,

$$D(t^{-1} \rightarrow 0) \sim v^2 P(0) \int_{t^{-1}} d\gamma \gamma^{-1} \sim \ln t / \rho_{\epsilon_F}. \quad (4)$$

Substituting it into the mean squared displacement, $\delta \mathbf{x}^2(t) \sim D(t^{-1} \rightarrow 0)t$, we reproduce Eq. (2). For a finite system, t^{-1} scales as $\sim L^{-2}$. In combination with the Einstein relation, Eq. (4) gives the conductance $g(L) \sim e^2 \ln L$. Therefore, we reproduce Eq. (3) also. Surprisingly, Eq. (4) is *insensitive* to the details of $P(\gamma)$. This universality may reflect the topological nature of the system.

We are now ready to present the analytical theory and some technical details of the proof. We begin with weakly disordered scalar potentials that are Gaussian distributed, $\overline{V(\mathbf{x})} = 0$, $\overline{V(\mathbf{x})V(\mathbf{x}')} = \frac{v^2}{\pi g_0} \delta(\mathbf{x} - \mathbf{x}')$. Here, the dimensionless parameter g_0 characterizes the disorder strength, and the overline stands for the disorder average. To proceed further we introduce the two-particle correlation function, $\phi^\mu(\mathbf{x} - \mathbf{x}', \omega) \equiv -\frac{1}{2\pi i} \text{tr} \sigma^\mu \overline{G_{\epsilon_F + \omega}^R(\mathbf{x}, \mathbf{x}') G_{\epsilon_F}^A(\mathbf{x}', \mathbf{x})}$, $\mu = 0, x, y$, where the matrix retarded (advanced) Green function obey $(\epsilon - \hat{H} \pm i\delta) G_\epsilon^{R,A}(\mathbf{x}, \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}') \sigma^0$. Notice that the disorder average restores the translational invariance and correlation functions thereby depend only on $\mathbf{x} - \mathbf{x}'$. As such, we may pass to the Fourier representation, $\overline{\phi^\mu(\mathbf{x} - \mathbf{x}', \omega)} \rightarrow \phi^\mu(\mathbf{q}, \omega) \equiv \phi^\mu(q)$ and likewise, $\overline{G_\epsilon^{R,A}(\mathbf{x}, \mathbf{x}')} \rightarrow \int d^2(\mathbf{x} - \mathbf{x}') e^{-i\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')} \overline{G_\epsilon^{R,A}(\mathbf{x}, \mathbf{x}')} \equiv \mathcal{G}_\epsilon^{R,A}(\mathbf{p})$. Here, $\mathcal{G}_\epsilon^{R,A}$ satisfies $(\epsilon - v\sigma^\alpha p_\alpha - \Sigma_\epsilon^{R,A}(\mathbf{p})) \mathcal{G}_\epsilon^{R,A}(\mathbf{p}) = \sigma^0$ [15]. For weak disorders, one may invoke the self-consistent Bonn approximation (SCBA) to calculate the self-energy, $\Sigma_\epsilon^{R,A}$, obtaining $\Sigma_\epsilon^{R,A} = \frac{v^2}{2\pi g_0} \sigma^0 \text{tr} \mathcal{G}_\epsilon^{R,A}$. (Notice that the trace operation includes the momentum index.) The density of states $\rho_\epsilon \equiv -\frac{1}{\pi} \text{tr} \text{Im} \mathcal{G}_\epsilon^R$ is related to the imaginary part of the self-energy via $\rho_\epsilon = -\frac{g_0}{v^2} \text{tr} \text{Im} \Sigma_\epsilon^R$. Importantly, the SCBA gives an exponentially small energy scale $\sim e^{-2\pi^2 g_0}$ [16]: for energies far below this scale (the Dirac regime), the density of states saturates, $\rho_\epsilon \sim e^{-2\pi^2 g_0}$, while for energies far above it, ρ_ϵ converges to its clean limit $\sim |\epsilon|$.

Then, by adaption of the method of Refs. [17, 18], it can be shown that in the hydrodynamic limit, $q \rightarrow 0$,

$$\omega \phi^0(q) - q_\alpha \phi^\alpha(q) = -\rho_{\epsilon_F}. \quad (5)$$

This is the macroscopic continuity equation reflecting the particle conservation law, where $\phi^0(q)$ is the macroscopic density and $(\phi^x(q), \phi^y(q))$ the macroscopic current. The latter follows a Fick-like law, read

$$\phi^\alpha(q) = -iD(q)q^\alpha\phi^0(q) \quad (6)$$

in the Fourier representation. Here, $D(q) \equiv v^2/(-i\omega + \gamma(q))$ is full quantum diffusion coefficient, where $\gamma(q)$ is the so-called “current relaxation kernel” given by

$$\gamma(q) = \frac{1}{\tau} + \frac{1}{2\pi\rho_{\epsilon_F}} \text{tr} \{ \hat{q}_\alpha \sigma^\alpha \Delta \mathcal{G}_q U_q \Delta \mathcal{G}_q \hat{q}_\beta \sigma^\beta \}. \quad (7)$$

Here, $\Delta \mathcal{G}_q(\mathbf{p}) \equiv \mathcal{G}_{\epsilon_F + \omega}^R(\mathbf{p} + \mathbf{q}) - \mathcal{G}_{\epsilon_F}^A(\mathbf{p})$, the elastic scattering rate $\frac{1}{\tau} = -2\text{trIm} \Sigma^R = v^2\rho_{\epsilon_F}/g_0$, and $U_q(\mathbf{p}, \mathbf{p}')$ is the two-particle irreducible vertex function. This equation, together with the expression of $D(q)$, builds up a bridge between the macroscopic hydrodynamics and the microscopic quantum dynamics. It is evident from Eq. (7) that localization effects – encoded by $U_q(\mathbf{p}, \mathbf{p}')$ – introduce renormalization of the elastic scattering rate. In the absence of quantum interference, Eq. (7) leads to a Boltzmann diffusion coefficient $D \equiv v^2\tau$ (with the classical transport mean free time twice larger than the elastic scattering time). Eqs. (5)-(7) constitute the framework of subsequent analysis.

The closed set of macroscopic equations (5) and (6) gives an exact relation between the optical conductivity, $\sigma(q)$, and the full quantum diffusion coefficient, $D(q)$. To this end, let us introduce the density response function defined as $\chi(\mathbf{x} - \mathbf{x}', t - t') \equiv i\langle [\Psi^\dagger(\mathbf{x}, t)\Psi(\mathbf{x}, t), \Psi^\dagger(\mathbf{x}', t')\Psi(\mathbf{x}', t')] \rangle$, with $\langle \cdot \rangle$ the ground state average. Upon passing to the Fourier representation, $\chi(\mathbf{x} - \mathbf{x}', t - t') \rightarrow \chi(q)$, one may follow the derivation of Ref. [17] to show $\chi(q) \stackrel{q \rightarrow 0}{\sim} \omega\phi^0(q) + \rho_{\epsilon_F}$. In combination with the solution to Eqs. (5) and (6), it gives $\chi(q) = \rho_{\epsilon_F} \frac{D(q)q^2}{-i\omega + D(q)q^2}$. As a result,

$$\sigma(q) \equiv -e^2 \lim_{\mathbf{q} \rightarrow 0} \left(\frac{i\omega}{\mathbf{q}^2} \chi(q) \right) = e^2 \rho_{\epsilon_F} D(q). \quad (8)$$

Thus, we justify the general Einstein relation for disordered Dirac particles.

The system’s behavior at large scales is determined by the current relaxation kernel, $\gamma(q)$. Therefore, we proceed to perform non-perturbative analysis of this kernel dominated by the infrared divergences of the two-particle irreducible vertex function $U_q(\mathbf{p}, \mathbf{p}')$. Generally, these divergences include two types: the singlet component of diffuson and cooperon [19]. It can be shown that the effective time-reversal symmetry leads to a mathematically rigorous theorem:

The current relaxation kernel $\gamma(q)$ suffers no diffuson-type infrared divergences.

It generalizes the Vollhardt-Wölfle theorem discovered for spinless electron systems [17].

Thanks to this theorem, to calculate the two-particle irreducible vertex, $U_q(\mathbf{p}, \mathbf{p}')$, we need to consider the diagrams composed of (the singlet component of) cooperon. For large frequencies, the dominant contribution to $U_q(\mathbf{p}, \mathbf{p}')$ consists of single cooperon, giving a quantum correction to the (bare) elastic scattering rate $\sim -\frac{1}{\pi\rho_{\epsilon_F}\tau} \int \frac{d^2\mathbf{k}}{(2\pi)^2} (-i\omega + D\mathbf{k}^2)^{-1}$ order of $\mathcal{O}(g_0^{-1})$. This is the well-known weak anti-localization, with the negative sign reflecting the destructive interference in the presence of the π Berry phase [10, 11, 19, 20]. It should be noted, however, that even for weak disorders, this quantum correction is divergent in the low-frequency limit $\omega \rightarrow 0$. In this case the elastic scattering rate (and thus the Boltzmann diffusion coefficient) undergoes strong renormalization and as such, the perturbative expansion in $1/g_0$ breaks down. To go beyond the perturbation, we must take into account the full two-particle irreducible vertex for Eq. (7). As a matter of the reciprocity principle, $U_q(\mathbf{p}, \mathbf{p}') \sim \phi^0(\mathbf{p} + \mathbf{p}', \omega) \sim [-i\omega + D(\omega)(\mathbf{p} + \mathbf{p}')^2]^{-1}$, where $D(\omega) \equiv D(\mathbf{q} = 0, \omega)$. This amounts to the replacement of the Boltzmann diffusion coefficient in the weak anti-localization correction by the dynamical diffusion coefficient $D(\omega)$. In doing so, we obtain a self-consistent equation of the dynamical diffusion coefficient,

$$\frac{D}{D(\omega)} = 1 - \frac{1}{\pi\rho_{\epsilon_F}} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{1}{-i\omega + D(\omega)\mathbf{k}^2}, \quad (9)$$

that is expected to hold for arbitrarily low frequencies. Eq. (9) implies the validity of one-parameter scaling in topological metals, consistent with non-perturbative studies based on field theories [14].

Let us discuss below the physical consequences of the general theory described by Eqs. (5)-(9).

a) Superdiffusion.—We prepare a wave packet with the energy near ϵ_F (> 0) and let it evolve. Its expansion is characterized by the full time-dependence of the mean squared displacement given by $\delta\mathbf{x}^2(t) = \int \frac{d\omega}{2\pi} (1 - e^{-i\omega t}) D(\omega)/\omega^2$. For low frequencies, $\omega\tau \ll e^{-4\pi^2 g_0}$, Eq. (9) may be solved analytically, giving $D(\omega)/D \approx \frac{1}{4\pi^2 g_0} \ln \frac{1}{-i\omega\tau}$. Substituting this result into the expression of $\delta\mathbf{x}^2(t)$, we obtain Eq. (2) for $t \gg \tau e^{4\pi^2 g_0}$. Noticing that $i\omega \sim t^{-1}$, we have $D(t^{-1})/D \approx \frac{1}{4\pi^2 g_0} \ln \frac{t}{\tau}$ namely Eq. (4). It is thereby justified that at large times, localization effects lead to strong renormalization of the elastic scattering rate.

b) Optical conductivity.—According to the Einstein relation (8), for low frequencies, $\omega\tau \ll e^{-4\pi^2 g_0}$, the optical conductivity reads $\sigma(\mathbf{q} = 0, \omega) \approx (\frac{e}{2\pi})^2 \ln \frac{1}{-i\omega\tau}$ that strikingly, is universal: it depends on the elastic scattering rate logarithmically.

c) Static conductance of a finite system.—We have focused on the dynamic property of a bulk (infinite extended) system so far. Now, we switch to the static con-

ductance of a finite system of size L . This subject is currently under intense investigations [2, 10–12]. In the static limit, Eq. (9) is simplified to

$$\frac{D}{D(0)} \left[1 + \frac{1}{\pi g_0} \int_{\frac{1}{L} < |\mathbf{k}| < \frac{1}{v\tau}} \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{1}{\mathbf{k}^2} \right] = 1 \quad (10)$$

thanks to $\frac{-i\omega}{D(\omega)} \xrightarrow{\omega \rightarrow 0} 0$. This equation gives the static diffusion coefficient $D(0) = D[1 + \frac{1}{2\pi^2 g_0} \ln \frac{L}{v\tau}]$.

To translate $D(0)$ to a static size-dependent conductance (identical to the conductivity in two dimensions), $g(L)$, we apply a static, infinitesimally small electric field E to the system say, along the x -direction. In response to the electric potential, $(L-x)eE$, a density profile is established across the sample, read $-eE \int_0^L dx' \chi(x-x', 0)(L-x') = eE \rho_{\epsilon_F}(x-L)$. In obtaining this result, we have used the fact that in the real space, the static density response function is $\chi(x-x', \omega=0) = \rho_{\epsilon_F} \delta(x-x')$. Results from this density profile a uniform diffusive current, $-e^2 \rho_{\epsilon_F} D(0) E$, that is balanced by a macroscopic electric current, $g(L)E$. It then follows $g(L) = e^2 \rho_{\epsilon_F} D(0)$, giving Eq. (3) (where the conductance is in unit of e^2/h .) For large samples, $L/v\tau \gg e^{2\pi^2 g_0}$, the conductance $g(L) = \frac{e^2}{2\pi^2} \ln \frac{L}{v\tau}$ that depends logarithmically on the elastic scattering rate and has a *universal* prefactor.

d) General disordered potentials.—Qualitatively, the macroscopic equations (5), (6) and (8) are the results of particle conservation, while Eq. (9) reflects the nature of one-parameter scaling and one-loop self-consistency. As such, we expect them to be universal for a large class disordered potential. Specifically, the disorder potentials – remaining of the scalar type – may be strong, i.e., $g_0 \lesssim 1$, and (or) exhibits long-ranged Gaussian correlations. These dramatic modifications notwithstanding, they merely affect the microscopic parameters namely ρ_{ϵ_F} and D in Eqs. (5), (6), (8) and (9). Then, we may repeat the discussions of *a)–c)*, obtaining Eqs. (2) and (3). The latter is consistent with numerical findings [2, 10–12], despite that the method here differs essentially from the routine one based on the Kubo formula [10].

In realistic electronic materials, electron-electron interaction interplays strongly with wave interference effects (as observed in experiments on topological insulators [7]). Therefore, we discuss here the possibility of observing superdiffusion in photonic materials that are free of such an interplay (Fig. 2, left). We adopt the method that leads to the experimental discovery of the so-called transverse localization [21]. In the present context, one may invoke the optical induction technique to fabricate a two-dimensional photonic crystal of thickness z that displays the honeycomb structure in the transverse (x - y) plane while is uniform in the longitudinal (z) direction. As such, the photonic band structure exhibits two non-equivalent Dirac valleys, around K and K' (Fig. 2, right). One may further introduce random fluctuations of

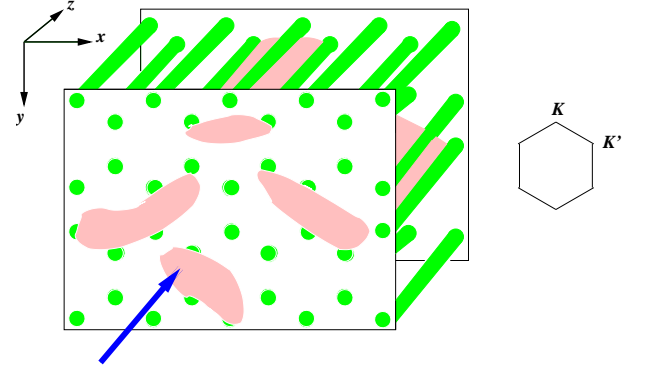


FIG. 2: Transverse superdiffusion. Left: a laser beam (blue) is launched into a two-dimensional disordered photonic crystal. The latter is uniform in the longitudinal (z) direction while in the transverse (x - y) plane, it displays a periodic background of honeycomb structure (green) with smooth random dielectric fluctuations (pink). At a sufficiently large distance z , the broadening of the probe beam scales as $\sim z \ln z$. Right: the first Brillouin zone of the honeycomb lattice.

refractive-index in the transverse plane that are smooth on a scale much larger than the lattice constant. Then, a monochromatic laser beam is launched into this disordered photonic crystal and undergoes diffraction broadening. At a distance z from the input plane, the intensity profile of the probe beam is measured.

The probe beam excites only the Bloch modes in certain Dirac valley, and has an initial width much larger than the scale over which the refractive-index fluctuates. Its propagation in the disordered photonic crystal is described by the (rescaled) paraxial Helmholtz equation [21], $i\partial_z A = [-\frac{1}{2}(\partial_x^2 + \partial_y^2) - n(x, y)]A$, where $A(x, y, z)$ is the envelop of the electric field smoothly varying in the longitudinal direction. Being static (having no-time dependence) notwithstanding, this equation possesses a perfect analogy to the Schrödinger equation, with z playing the role of “time” and the (negative of) total refractive-index, $-n(x, y)$, of “potential”. As a result, the diffraction broadening of the probe beam, $A(x, y, z)$, mimics quantum wave function at time z . Because the “potential” is smooth enough, the inter-valley scattering is suppressed. Effectively, this “quantum dynamics” is reduced to that described by Eq. (1), with the spin degree of freedom accounts for the sublattice structure. Thus, we expect that the broadening of the probe beam (in the transverse plane), $\iint dx dy (x^2 + y^2) |A(x, y, z)|^2$, scales as $\sim z \ln z$ for sufficiently large thickness z . We term this phenomenon *transverse superdiffusion*.

In summary, we develop a non-perturbative theory of large-scale quantum dynamics of Dirac particles in disordered environments. We show that for general disorder strength and carrier doping, (i) a bulk topological metal exhibits superdiffusion (2) at large times; and (ii) the static conductance of a finite-size system follows the

scaling equation (3), consistent with the numerical results [2, 10–12]. We discuss the ensuing consequence – the transverse superdiffusion in two-dimensional photonic materials – that might be within the current experimental reach.

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- [1] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, *Rev. Mod. Phys.* **81**, 109 (2009).
 - [2] S. Das Sarma, S. Adam, E. H. Hwang, and E. Rossi, *Rev. Mod. Phys.* **83**, 407 (2011); E. R. Mucciolo and C. H. Lewenkopf, *J. Phys.: Condens. Matter* **22**, 273201 (2010), and references therein.
 - [3] M. Z. Hasan and C. L. Kane, *Rev. Mod. Phys.* **82**, 3045 (2010); X. L. Qi and S. C. Zhang, *ibid.* **83**, 1057 (2011).
 - [4] L. Fu and C. L. Kane, *Phys. Rev. B* **76**, 045302 (2007); J. L. Moore and L. Balents, *ibid.* **75**, 121306 (R) (2007).
 - [5] H. J. Zhang, *et. al.* *Nat. Phys.* **5**, 438 (2009); *Nature* **452**, 970 (2008); Y. Xia, *et. al.* *Nat. Phys.* **5**, 398 (2009); Y. L. Chen, *et. al.* *Science* **325**, 178 (2009).
 - [6] M. I. Katsnelson, K. S. Novoselov, and A. K. Geim, *Nat. Phys.* **2** 620 (2006); V. V. Cheianov and V. I. Fal’ko, *Phys. Rev. B* **74**, 041403 (2006).
 - [7] See e.g., M. Liu, *et. al.* *Phys. Rev. B* **83**, 165440 (2011).
 - [8] O. Peleg, *et. al.* *Phys. Rev. Lett.* **98**, 103901 (2007).
 - [9] M. J. Ablowitz, S. D. Nixon, and Y. Zhu, *Phys. Rev. A* **79**, 053830 (2009).
 - [10] K. Nomura, M. Koshino, and S. Ryu, *Phys. Rev. Lett.* **99**, 146806 (2007).
 - [11] J. H. Bardarson, J. Tworzydło, P.W. Brouwer, and C.W. J. Beenakker, *Phys. Rev. Lett.* **99**, 106801 (2007).
 - [12] S. Adam, P. W. Brouwer, and S. Das Sarma, *Phys. Rev. B* **79**, 201404 (R) (2009); S. Adam, E. H. Hwang, V. M. Galitski, and S. Das Sarma, *Proc. Natl. Acad. Sci. USA* **104**, 18392 (2007).
 - [13] J. P. Bouchaud and P. Le Doussal, *J. Stat. Phys.* **41**, 225 (1985); J. P. Bouchaud and A. Georges, *Phys. Rept.* **195**, 127 (1990).
 - [14] P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, *Phys. Rev. Lett.* **98**, 256801 (2007); S. Ryu, C. Mudry, H. Obuse, and A. Furusaki, *ibid.* **99**, 116601 (2007).
 - [15] Throughout this Letter the Greek indices α, β run over x, y and the Einstein convention is implied.
 - [16] A. Schuessler, P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, *Phys. Rev. B* **79**, 075405 (2009).
 - [17] D. Vollhardt and P. Wölffe, *Phys. Rev. B* **22**, 4666 (1980); D. Vollhardt and P. Wölffe, in *Electronic Phase Transitions*, edited by W. Hanke and Yu. V. Kopayev (North-Holland, Amsterdam, 1992).
 - [18] H. T. Nieh, P. Sheng, and X. B. Wang, *Phys. Lett. A* **246**, 542 (1998).
 - [19] S. Hikami, A. I. Larkin, and Y. Nagaoka, *Prog. Theor. Phys.* **63**, 707 (1980).
 - [20] T. Ando and T. Nakanishi, *J. Phys. Soc. Jpn.* **67**, 1704 (1998); T. Ando, T. Nakanishi, and R. Saito, *ibid.* **67**, 2857 (1998).
 - [21] T. Schwartz *et al.*, *Nature (London)* **446**, 52 (2007); H. De Raedt, A. Lagendijk, and P. de Vries, *Phys. Rev. Lett.* **62**, 47 (1989).